$$
\begin{array}{r|rrrr|l}
 & 1 & -1 & 1 & -3 & -1 \\
1 & -1 & 2 & -3 & 4 & 2 \\
2 & -2 & 1 & 2 & 1 & \alpha+2 \\
-4 & 4 & 1 & -10 & 1 & -3 \\
\hline
 & 1 & -1 & 1 & -3 & 1 \\
1 & 0 & 1 & -2 & 1 & 1 \\
-5 & 0 & -1 & 4 & -5 & \alpha \\
 & 0 & 5 & -14 & 13 & 1 \\
\hline
 & 1 & -1 & 1 & -3 & -1 \\
 & 0 & 1 & -2 & 1 & 1 \\
2 & 0 & 0 & 2 & -4 & \alpha+1 \\
 & 0 & 0 & -4 & 8 & -4 \\
\hline
 & 1 & -1 & 1 & -3 & -1 \\
 & 0 & 1 & -2 & 1 & 1 \\
 & 0 & 0 & 2 & -4 & \alpha+1 \\
 & 0 & 0 & 0 & 0 & 2\alpha-2
\end{array}
$$

~ 8 min

$\downarrow$

$\boxed{\text{~ 20 min pour les étudiants}}$

$$\boxed{\text{Sol. } \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} = \begin{pmatrix} 1 \\ 3 \\ 1 \\ 0 \end{pmatrix} + \lambda \begin{pmatrix} 4 \\ 3 \\ 2 \\ 1 \end{pmatrix}}$$

$\boxed{\nexists \text{ sol. si } \alpha \neq 1}$. Soit $\alpha = 1$:

$\underline{x_4 := \lambda}$ $\quad 2x_3 - 4\lambda = 2 \Rightarrow \underline{x_3 = 1 + 2\lambda}$

$x_2 - 2x_3 + x_4 = 1 \Rightarrow \underline{x_2} = 2(1+2\lambda) - \lambda + 1 = \underline{3 + 3\lambda}$

$\underline{x_1} = x_2 - x_3 + 3x_4 - 1 = \underline{1 + 4\lambda}$

$$
\begin{array}{r|rrrr|r}
 & 1 & 1 & -1 & -1 & 1 \\
-2 & 2 & 3 & -1 & -3 & 4 \\
2 & -2 & 2 & 5 & -4 & \alpha+4 \\
-3 & 3 & 2 & -2 & 2 & 3 \\
\hline
 & 1 & 1 & -1 & -1 & 1 \\
 & 0 & 1 & 1 & -1 & 2 \\
-4 & 0 & 4 & 3 & -6 & \alpha+6 \\
1 & 0 & -1 & +1 & 5 & 0 \\
\hline
 & 1 & 1 & -1 & -1 & 1 \\
 & 0 & 1 & 1 & -1 & 2 \\
 & 0 & 0 & -1 & -2 & \alpha-2 \\
2 & 0 & 0 & 2 & 4 & 2 \\
\hline
 & 1 & 1 & -1 & -1 & 1 \\
 & 0 & 1 & 1 & -1 & 2 \\
 & 0 & 0 & -1 & -2 & \alpha-2 \\
 & 0 & 0 & 0 & 0 & 2\alpha-2
\end{array}
$$

~ 8 min

$\boxed{\nexists \text{ sol. si } \alpha \neq 1}$

Soit $\underline{\alpha = 1}$:

sol

$$\begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \\ 1 \\ 0 \end{pmatrix} + \lambda \begin{pmatrix} -4 \\ 3 \\ -2 \\ 1 \end{pmatrix}$$

$x_4 = \lambda \quad , \quad x_3 = -2\lambda + 1$

$x_2 = -x_3 + x_4 + 2 = 3\lambda + 1$

$x_1 = -x_2 + x_3 + x_4 + 1 = -4\lambda + 1$