Sol Ex

Pendant le CM 11
Fiche de TD __, Ex. 41

20/04
Gricey
ALGÈBRE 2

orth. ⇒ $\langle \cdot | \cdot \rangle = 0$

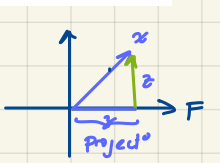
colinéaires : exp. en comme mult. de l'autre.
$\left\langle \begin{pmatrix}1\\1\end{pmatrix}, \begin{pmatrix}-1\\1\end{pmatrix} \right\rangle = -1 + 1 = 0$

Exercice 41. Trouver la matrice M de l'exercice 27.4 avec la méthode de l'exercice précédent.

$p : \mathbb{R}^2 \longrightarrow \mathbb{R}^2$, $x \longmapsto Mx$

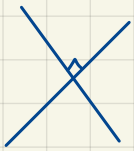
Projet° sur F // G.

$\mathcal{E} = \left(\begin{pmatrix}1\\0\end{pmatrix}, \begin{pmatrix}0\\1\end{pmatrix}\right)$ (e_1, e_2) — Base can. de $\mathbb{R}^2$



Proj. sur F // G base

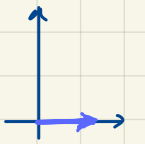
$F = \text{vect}\left(\begin{pmatrix}2\\1\end{pmatrix}\right)$, $G = \text{vect}\left(\begin{pmatrix}3\\\alpha\end{pmatrix}\right)$



$\left\langle \begin{pmatrix}2\\1\end{pmatrix}, \begin{pmatrix}3\\\alpha\end{pmatrix} \right\rangle = 0$
$\Rightarrow 2\times 3 + 1\times\alpha = 0$
$\Rightarrow \alpha = -6$

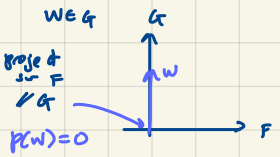
$G = \text{vect}\left(\begin{pmatrix}3\\-6\end{pmatrix}\right)$

$v \in F$
$p(v) =$



sur F // G : ssi $\vec{0}$

$w \in G$



$p(w) = 0$

dans $\mathcal{E}$

$\begin{cases} M \cdot v = v \\ M \cdot w = 0 \end{cases}$ pour $v \in F$, pour $w \in G$

$\mathcal{B} = \left\{ \begin{pmatrix}2\\1\end{pmatrix}, \begin{pmatrix}3\\-6\end{pmatrix} \right\}$
f_1 f_2

$f_1[\mathcal{E}] = \begin{pmatrix}2\\1\end{pmatrix}$ $f_{1[\mathcal{B}]} = \begin{pmatrix}1\\0\end{pmatrix}$
$f_2[\mathcal{E}] = \begin{pmatrix}3\\-6\end{pmatrix}$ $f_2[\mathcal{B}] = \begin{pmatrix}0\\1\end{pmatrix}$

dans $\mathcal{B}$

$\begin{cases} M\begin{pmatrix}1\\0\end{pmatrix} = \begin{pmatrix}1\\0\end{pmatrix} \\ M\begin{pmatrix}0\\1\end{pmatrix} = \begin{pmatrix}0\\0\end{pmatrix} \end{cases}$

$\begin{cases} \begin{pmatrix}a & b\\c & a\end{pmatrix}\begin{pmatrix}1\\0\end{pmatrix} = \begin{pmatrix}1\\0\end{pmatrix} \\ \begin{pmatrix}a & b\\c & d\end{pmatrix}\begin{pmatrix}0\\1\end{pmatrix} = \begin{pmatrix}0\\0\end{pmatrix} \end{cases} \Rightarrow \begin{cases} \begin{pmatrix}a\\c\end{pmatrix} = \begin{pmatrix}1\\0\end{pmatrix} \\ \begin{pmatrix}b\\d\end{pmatrix} = \begin{pmatrix}0\\0\end{pmatrix} \end{cases} \Rightarrow \begin{matrix} a = 1 \\ c = 0 \\ b = 0 \\ d = 0 \end{matrix}$

$M_{[\mathcal{B}]} = \begin{pmatrix}1 & 0\\0 & 0\end{pmatrix}$

base $\mathcal{B}$: p_1, p_2 ; base $\mathcal{E}$: e_1, e_2

$P = \begin{pmatrix}2 & 3\\1 & -6\end{pmatrix}$

$P^{-1} = \frac{1}{-15}\begin{pmatrix}-6 & -3\\-1 & 2\end{pmatrix} = \frac{1}{15}\begin{pmatrix}6 & 3\\1 & -2\end{pmatrix}$

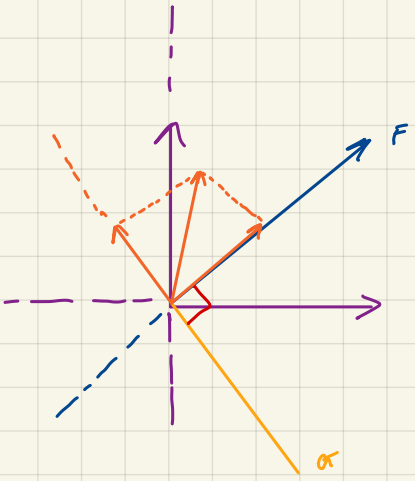
$-12 - 3 = -15$

$M_{[\mathcal{E}]} = P\, M_{[\mathcal{B}]}\, P$

$\mathcal{E}\left(\#\right)\left(\ \right)_{\mathcal{B}}$ — $\mathcal{B}$ $\mathcal{B}$

$\mathcal{M}_{n,m}(\mathbb{R}) \times \mathcal{M}_{\mathcal{B},\mathcal{B}}(\mathbb{R}) \times \mathcal{M}_{\mathcal{E},\mathcal{E}}(\mathbb{R})$

$M_{[\mathcal{E}]} = P\, M_{[\mathcal{B}]}\, P^{-1}$

$= \begin{pmatrix}2 & 3\\1 & -6\end{pmatrix}\begin{pmatrix}1 & 0\\0 & 0\end{pmatrix}\frac{1}{15}\begin{pmatrix}6 & 3\\1 & -2\end{pmatrix}$

$= \begin{pmatrix}2 & 0\\1 & 0\end{pmatrix}\frac{1}{15}\begin{pmatrix}6 & 3\\1 & -2\end{pmatrix} = \frac{1}{15}\begin{pmatrix}12 & 6\\6 & 3\end{pmatrix}$