Sol Ex

Pendant le CM 9
Fiche de TD 5, Ex. 12

28/03
Lucey
ANALYSE 2

a) Résoudre $I(x) = \int_0^x \frac{2}{1-u^2}\,du = \int_0^x \frac{2}{(1-u)(1+u)}\,du$ (*)

DES:

$$\frac{A}{1-u} + \frac{B}{1+u} \Rightarrow A = B = 1$$

$$(*) = \int_0^x \frac{1}{1+u} + \frac{1}{1-u}\,du \qquad (\ln(1-u))' = \frac{-1}{1-u}$$

$$= \left[-\ln(|1-u|) + \ln(|1+u|)\right]_0^x \qquad x \in]-1,1[$$

$$= -\ln(1-x) + \ln(1+x)$$

$$= \ln\left(\frac{1+x}{1-x}\right) \qquad |u| \leq |x| < 1$$

b) Résoudre $J(x) = \int_0^x \frac{2}{\cos(v)}\,dv = 2\int_0^x \frac{\cos(v)}{1-\sin^2(v)}\,dv$

$\cos^2 v = 1 - \sin^2 v$

$u = \sin(v) \qquad x \in [0,x] \subset [0, \pm\frac{\pi}{2}[$

$\Rightarrow v = \arcsin(u)$; $\cos(v) = \cos(\arcsin(u)) = \sqrt{1-u^2}$

$dv = \frac{1}{\sqrt{1-u^2}}\,du$

$$= 2\int_0^{\sin(x)} \frac{1}{1-u^2} \cdot \frac{\sqrt{1-u^2}}{\sqrt{1-u^2}}\,du = I(\sin(x))$$

($\cos(v)$; $du \to dv$)

c) Trouver sol. gén. $\cos(x)\cdot y' - 2y = 0 \qquad \cos(x) \neq 0$

$$\Leftrightarrow y'(x) - \frac{2}{\cos(x)} \cdot y(x) = 0$$

$$\Rightarrow y(x) + a(x)\cdot y(x) = 0$$

$$\Rightarrow y_h(x) = C \cdot e^{-\int_0^x a(v)\,dv} \qquad C \in \mathbb{R}$$

$$= C\, e^{I(\sin(x))}$$

$$= C \cdot e^{\ln\left(\frac{1+\sin(x)}{1-\sin(x)}\right)}$$

$$= C \cdot \frac{1+\sin(x)}{1-\sin(x)}$$

d) Trouver une sol. part. de $\cos(x)\,y' - 2y = 2\cos(x) + \sin(2x)$

$y_p(x) = C(x)\cdot y_h(x)$

$y_p'(x) = C'(x)\cdot y_h(x) + C(x)\cdot y_h'(x)$

$\cos(x)\cdot y_p'(x) - 2y_p(x) = 2\cdot\cos(x) + \sin(2x)$

$\Rightarrow \cos(x)\left[C'(x)\cdot y_h(x) + C(x)\cdot y_h'(x)\right] - 2C(x)\cdot y_h(x) = 2\cos(x) + \sin(2x)$

$\Rightarrow \cos(x)\cdot C'(x)\cdot y_h(x) + C(x)\left[\cos(x)\cdot y_h(x) - 2y_h(x)\right] = 2\cos(x) + \sin(2x) \Rightarrow$

($=0$ car y_h solution de c))

$$\Rightarrow C'(x) = \frac{2\cos(x) + \sin(2x)}{\cos(x)\cdot y_h(x)} = \frac{2\cos(x) + 2\cos(x)\cdot\sin(x)}{\cos(x)\cdot y_h(x)}$$

Double angle formula : $\sin(2x) = 2\cos(x)\cdot\sin(x)$

$$C'(x) = 2 \cdot \frac{1+\sin(x)}{\frac{1+\sin(x)}{1-\sin(x)}} = 2(1-\sin(x))$$

$$C(x) = 2(x+\cos x)$$

$$\Rightarrow y_p(x) = C(x) y_1(x) = 2(x+\cos x) \cdot \frac{1+\sin(x)}{1-\sin(x)}$$

e) Conclure

$$y_G(x) = y_1(x) + y_p(x)$$
$$= C \cdot \frac{1+\sin x}{1-\sin x} + 2(x+\cos x) \cdot \frac{1+\sin(x)}{1-\sin(x)}$$

$$y_G(0) = 1$$

$$\Rightarrow C + 2(0+1) = 1$$
$$\Rightarrow C + 2 = 1 \quad \Rightarrow C = -1$$

$$y_G(x) = -\frac{1+\sin(x)}{1-\sin(x)} + 2(x+\cos(x)) \cdot \frac{1+\sin(x)}{1-\sin(x)} \iff$$

$$\iff -\frac{1+\sin(x)}{1-\sin(x)} + (2x - 1 + 2\cos(x))$$

plus détaillé

$$\iff \frac{-1+\sin x}{1+\sin x} + \frac{2x+2\cos x+\sin x-1}{1-\sin x}$$

$$\iff \frac{-1+\sin x + 2x+2\cos x+\sin x-1}{1-\sin x}$$

$$\iff \frac{-1+\sin x+2x+2\cos x+\sin x-1}{1-\sin x}$$

$$\iff \frac{2x+2\cos x+\sin x-1}{1-\sin x} = \frac{2x-1+2\cos x}{1-\sin x}$$