Sol Ex

Pendant le CM 12
~~Fiche de TD~~ ___, ~~Ex.~~ CC3 sujet 1

25/04
Gricey
ANALYSE 2

Exo: 1

$y'' + 2y' + 2y = 10e^x$
$y(0) = 1$, $y'(0) = 2$

a) $y'' + 2y' + 2y = 0$

éq. caract. $\lambda^2 + 2\lambda + 2 = 0$

$\Delta = 4 - 8 = -4 < 0$

$\lambda = \frac{-2 + i\sqrt{4}}{2} = -1 + i$

$\bar{\lambda} = -1 - i$

$y_H(x) = C_1 e^{-x} \cos(x) + C_2 e^{-x} \sin(x)$

b) Soluto part. de $y'' + 2y' + 2y = 10e^x$

$y(x) = a \cdot e^x$, $a \in \mathbb{R}$
$y'(x) = a \cdot e^x$
$y''(x) = a \cdot e^x$

$ae^x + 2ae^x + 2ae^x = 10e^x$
$5a = 10$
$a = 2$

Une soluto part. est
$y_p(x) = 2e^x$

c) $y(x) = 2e^x + C_1 e^{-x} \cdot \cos(x) + C_2 e^{-x} \sin(x)$

$y'(x) = 2e^x + C_1 e^{-x} \cos(x) - C_1 e^{-x} \sin(x) - C_2 e^{-x} \sin(x) + C_2 e^{-x} \cos(x)$

$\begin{cases} y(0) = 1 \\ y'(0) = 2 \end{cases}$

$\Leftrightarrow \begin{cases} y(0) = 2 + C_1 = 1 \\ y'(0) = 2 - C_1 + C_2 = 1 \end{cases}$

$\Leftrightarrow \begin{cases} C_1 = -1 \\ C_2 = -1 \end{cases}$

$y(x) = 2e^x - e^{-x} \cos(x) - 2^{-x} \sin(x)$

Exo: 2

$(1+x^2)y' + 2xy = x\cos x$
$y(0) = 0$

1) (H): $(1+x^2)y' + 2xy = 0$

$y_H' = \frac{-2x}{(1+x^2)} y_H$

$y_H(x) = Ke^{A(x)}$ avec $A(x)$ primitive de $a(x) = \frac{-2x}{1+x^2}$

$\int \frac{-2x}{1+x^2} dx = -\ln(1+x^2)$

$\frac{u'}{u}$

$y_H(x) = K \cdot e^{-\ln(1+x^2)} = K e^{\ln\left(\frac{1}{1+x^2}\right)} = \frac{K}{1+x^2}$

Solution de (H) est
$y_H(x) = \frac{K}{1+x^2}$ $K \in \mathbb{R}$

2) $y_p(x) = \frac{K(x)}{1+x^2}$

$y_p'(x) = \frac{K'(x)(1+x^2) - K(x)2x}{(1+x^2)^2}$

$(1+x^2) y_p'(x) + 2x y_p(x) = x\cos x$

$\Leftrightarrow \cancel{(1+x^2)} \frac{K'(x)(1+x^2) - K(x)2x}{(1+x^2)^{\cancel{2}}} + 2x \frac{K(x)}{(1+x^2)} = x \cdot \cos x$

$\Leftrightarrow K'(x) - \cancel{\frac{K(x)2x}{1+x^2}} + 2x \cancel{\frac{K(x)}{1+x^2}} = x \cdot \cos x$

$\Leftrightarrow K'(x) = x \cdot \cos(x)$

$K'(x) = x \cdot \cos x$

$K(x) = \int x \cos(x)\, dx$

$v(x) = x$
$u'(x) = \cos(x) \longrightarrow u(x) = \sin(x)$

$= x \sin(x) - \int 1 \times \sin(x)\, dx$

$= x \sin(x) + \cos(x)$

$y_p(x) = \frac{K(x)}{1+x^2} = \frac{x\sin(x) + \cos(x)}{1+x^2}$

3) $y(x) = y_H(x) + y_p(x)$

$= \frac{K}{1+x^2} + \frac{x\sin(x) + \cos(x)}{1+x^2}$ $K \in \mathbb{R}$

$= \frac{K + x\sin(x) + \cos(x)}{1+x^2}$

$y(0) = 0$

$\frac{K + \cancel{0 \times \sin(0)} + \cos(0)}{1+0^2} = \frac{K+1}{1} = K+1$

$y(0) = 0$

$\int x^2 \cos(x)\, dx$

$x^2 \sin(x) - \int 3x^2 \sin(x)\, dx$

$-3x^2 \cos(x) + \int 6x\cos(x)\, dx$