Sol Ex

Pendant le CM 11
Fiche de TD 5, Ex. 2 (c), d), f)) | 24 (a), b)) | 25

Exo: 2 $x \in]-\frac{1}{2}, \frac{1}{2}[$

c) $\sqrt{1-4x^2}\, y' + 4xy = 0$

$\Rightarrow y' + \frac{4x}{\sqrt{1-4x^2}} y = 0$

$\Rightarrow y' + a(x)y = 0$

$\frac{1}{\sqrt{1-4x^2}} = \frac{-8x}{2\sqrt{1-4x^2}} = \frac{-4x}{\sqrt{1-4x^2}}$

$y(x) = Ce^{-\int a(x)\,dx}$ $C \in \mathbb{R}$

$= Ce^{-[-\sqrt{1-4x^2}]}$

$= Ce^{\sqrt{1-4x^2}}$

$y(0) = 1 \Rightarrow C \cdot e^1 = 1$

$\Rightarrow C = e^{-1}$

$y(x) = e^{\sqrt{1-4x^2}-1}$

d) $(1-4x^2)y' + 4xy = 0$

$y' + \frac{4x}{1-4x^2} y = 0$

$\Rightarrow y' + a(x)y = 0$

$(-\frac{1}{2})(\ln(1-4x^2))' = \frac{-8x}{1-4x^2} \times (-\frac{1}{2})$

$= \frac{4x}{1-4x^2}$

$y(x) = C \cdot e^{-\int a(x)\,dx}$ $C \in \mathbb{R}$

$= C \cdot e^{\frac{1}{2}\ln(1-4x^2)}$

$= C \cdot e^{\ln(\sqrt{1-4x^2})}$

$= C\sqrt{1-4x^2}$

$y(0) = 1$

$\Rightarrow C = 1$

$y(x) = \sqrt{1-4x^2}$

e) $(1-4x^2)y' + 8xy = 0$

$\Rightarrow y' + \frac{8x}{1-4x^2} y = 0$

$(\ln(1-4x^2))' = \frac{-8x}{1-4x^2}$

$\Rightarrow y(x) = C \cdot e^{-\int a(x)\,dx}$ $1 \in \mathbb{R}$

$= C \cdot e^{\ln(1-4x^2)}$

$= C(1-4x^2)$

$y(0) = 1$

$\Rightarrow C = 1$

$y(x) = 1-4x^2$

f) $(1-4x^2)y' + 16xy = 0$

$y' + \frac{16x}{1-4x^2} y = 0$

$(-2)(\ln(1-4x^2))' = \frac{-8x}{1-4x^2}(-2)$

$= \frac{16}{1-4x^2}$

$\Rightarrow y(x) = C \cdot e^{-\int a(x)\,dx}$

$= C \cdot e^{2\ln(1-4x^2)}$

$= C \cdot e^{\ln((1-4x^2)^2)}$

$= C(1-4x^2)^2$

$y(0) = 1 \Rightarrow C = 1$

$\Rightarrow y(x) = (1-4x^2)^2$ □.

Exo: 24 $y'' - 3y' + 2y = 4x^2$

a) $y_p(x) = ax^2 + bx + c$

$y_p'(x) = 2ax + b$, $y_p''(x) = 2a$

$2a - 3(2ax+b) + 2(ax^2+bx+c) = 4x^2$

$\Leftrightarrow 2ax^2 + (2b-6a)x + (2x+2a-3b) = 4x^2$

$\Rightarrow \begin{cases} 2a = 4 \\ 2b-6a = 0 \\ 2c+2a-3b = 0 \end{cases} \Rightarrow \begin{cases} a = 2 \\ 2b-12 = 0 \\ 4-3b+2c = 0 \end{cases} \Rightarrow \begin{cases} a = 2 \\ b = 6 \\ 2c = -4+18 = 14 \end{cases} \Rightarrow \begin{cases} a = 2 \\ b = 6 \\ c = 7 \end{cases}$

$y_p(x) = 2x^2 + 6x + 7$

b) (H): $y'' + 3y' + 2y = 0$

Équation caractéristique

$\lambda^2 - 3\lambda + 2 = 0$

$\Delta = 9 - 8 = 1 > 0$

$\lambda_1 = \frac{3-1}{2} = 1$, $\lambda_2 = \frac{3+1}{2} = 2$

$y_h(x) = C_1 e^{\lambda_1 x} + C_2 e^{\lambda_2 x}$, $C_1, C_2 \in \mathbb{R}$

$= C_1 e^x + C_2 e^{2x}$

$y_G(x) = y_h(x) + y_p(x)$

$= C_1 \cdot e^x + C_2 \cdot e^{2x} + 2x^2 + 6x + 7$ □.

Exo: 25

$y'' + 4y = \cos x$

(H): $y'' + 4y = 0$, $ay'' + \underbrace{by'}_{=0} + cy = 0$

Eq. caractéristique

$\lambda^2 + 4 = 0$

$\Rightarrow \lambda^2 = -4$

$\Rightarrow \lambda_1 = 2i$, $\lambda_2 = -2i$

$\lambda_{1,2} = \pm 2i$

$y_h(x) = C_1 \cos(2x) + C_2 \sin(2x)$ $C_1, C_2 \in \mathbb{R}$

$\sin(-2x)$

$= -\sin(2x)$ □.