Sol Ex

Pendant le CM 8

Fiche de TD 2, Ex. 24

16/03

Gricey

ALGÈBRE 2

Ex:

a) $F = \{(x,y,z,t) \in \mathbb{R}^4 \text{ t.q. } \quad +z+t=0\}$

$t = -x-y-z$

$$\begin{pmatrix} x \\ y \\ z \\ t \end{pmatrix} \in F \iff \begin{pmatrix} x \\ y \\ z \\ t \end{pmatrix} = \begin{pmatrix} x \\ y \\ z \\ -x-y-z \end{pmatrix} = x\begin{pmatrix} 1 \\ 0 \\ 0 \\ -1 \end{pmatrix} + y\begin{pmatrix} 0 \\ 1 \\ 0 \\ -1 \end{pmatrix} + z\begin{pmatrix} 0 \\ 0 \\ 1 \\ -1 \end{pmatrix}$$

$$F = \mathrm{Vect}\left(\begin{pmatrix} 1 \\ 0 \\ 0 \\ -1 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ 0 \\ -1 \end{pmatrix}, \begin{pmatrix} 0 \\ 0 \\ 1 \\ -1 \end{pmatrix}\right), \quad \dim F = 3$$

$G = \{(x,y,z,t) \in \mathbb{R}^4 \text{ t.q. } x+y = z+t\}$

$t = x+y-z$

$$\begin{pmatrix} x \\ y \\ z \\ t \end{pmatrix} \in G \iff \begin{pmatrix} x \\ y \\ z \\ t \end{pmatrix} = \begin{pmatrix} x \\ y \\ z \\ x+y-z \end{pmatrix} = x\begin{pmatrix} 1 \\ 0 \\ 0 \\ 1 \end{pmatrix} + y\begin{pmatrix} 0 \\ 1 \\ 0 \\ 1 \end{pmatrix} + z\begin{pmatrix} 0 \\ 0 \\ 1 \\ -1 \end{pmatrix}$$

$$G = \mathrm{Vect}\left(\begin{pmatrix} 1 \\ 0 \\ 0 \\ 1 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ 0 \\ 1 \end{pmatrix}, \begin{pmatrix} 0 \\ 0 \\ 1 \\ -1 \end{pmatrix}\right), \quad \dim G = 3$$

$$F \oplus G = \mathbb{R}^4 \iff \begin{array}{l} F+G = \mathbb{R}^4 \\ F \cap G = \{0_{\mathbb{R}^4}\} \end{array}$$

Soit $u = \begin{pmatrix} x \\ y \\ z \\ -x-y-z \end{pmatrix} = \begin{pmatrix} x \\ y \\ z \\ x+y-z \end{pmatrix} \iff -x-y-z = x+y-z \iff -x-y = x+y \iff x = -y$

$u = \begin{pmatrix} 1 \\ -1 \\ 0 \\ 0 \end{pmatrix} \in F \cap G \neq 0_{\mathbb{R}^4}$ pour $x = 1$

F et G ne sont pas en s. directe.

Soit $u = \begin{pmatrix} x \\ y \\ z \\ -x-y-z \end{pmatrix} = \begin{pmatrix} x \\ x \\ x \\ x \end{pmatrix} \iff \begin{cases} y = x \\ z = x \\ -x-y-z = x \end{cases} \iff \begin{cases} y = x \\ z = x \\ -4x = 0 \end{cases} \iff \begin{cases} y = 0 \\ z = 0 \\ x = 0 \end{cases}$

$u = 0_{\mathbb{R}^4}$ e $F \cap H = 0_{\mathbb{R}^4}$ et $F \oplus H = \mathbb{R}^4$

F et H sont en s. directe.

$H = \{(a,b,c,d) \in \mathbb{R}^4 \text{ t.q. } a=b=c=d\}$

$$\begin{pmatrix} a \\ b \\ c \\ d \end{pmatrix} \in H \iff \begin{pmatrix} a \\ b \\ c \\ d \end{pmatrix} = \begin{pmatrix} a \\ a \\ a \\ a \end{pmatrix} = a \cdot \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \end{pmatrix}$$

$H = \mathrm{Vect}\left(\begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \end{pmatrix}\right)$ dim $H = 1$

b) $$F+G = \mathrm{Vect}\left(\begin{pmatrix} 1 \\ 0 \\ 0 \\ -1 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ 0 \\ -1 \end{pmatrix}, \begin{pmatrix} 0 \\ 0 \\ 1 \\ -1 \end{pmatrix}, \begin{pmatrix} 1 \\ 0 \\ 0 \\ 1 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ 0 \\ 1 \end{pmatrix}, \cancel{\begin{pmatrix} 0 \\ 0 \\ 1 \\ -1 \end{pmatrix}}\right)$$

$\dim F+G \leq 4$, Soient $\lambda_1, \dots, \lambda_5 \in \mathbb{R}$

$$\lambda_1\begin{pmatrix} 1 \\ 0 \\ 0 \\ -1 \end{pmatrix} + \lambda_2\begin{pmatrix} 0 \\ 1 \\ 0 \\ -1 \end{pmatrix} + \lambda_3\begin{pmatrix} 0 \\ 0 \\ 1 \\ -1 \end{pmatrix} + \lambda_4\begin{pmatrix} 1 \\ 0 \\ 0 \\ 1 \end{pmatrix} + \lambda_5\begin{pmatrix} 0 \\ 1 \\ 0 \\ 1 \end{pmatrix} + \lambda_6\begin{pmatrix} 0 \\ 0 \\ 1 \\ -1 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

$$\Rightarrow \begin{cases} \lambda_1 + \lambda_4 = 0 \\ \lambda_2 + \lambda_5 = 0 \\ \lambda_3 = 0 \\ -\lambda_1 - \lambda_2 - \cancel{\lambda_3} + \lambda_4 + \lambda_5 = 0 \end{cases} \Rightarrow \begin{cases} \lambda_1 = -\lambda_4 \\ \lambda_2 = -\lambda_5 \\ 2\lambda_5 + 2\lambda_4 = 0 \\ \lambda_3 = 0 \end{cases} \Rightarrow \begin{cases} \lambda_1 = \lambda_5 \\ \lambda_2 = -\lambda_5 \\ \lambda_4 = -\lambda_5 \\ \lambda_3 = 0 \end{cases}$$

F + G : ces vecteurs st linéairemt indépendants, $\dim F+G = 4 \subseteq \mathbb{R}^4$. Soit $F+G = \mathbb{R}^4$.

$$F+H = \mathrm{Vect}\left(\begin{pmatrix} 1 \\ 0 \\ 0 \\ -1 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ 0 \\ -1 \end{pmatrix}, \begin{pmatrix} 0 \\ 0 \\ 1 \\ -1 \end{pmatrix}, \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \end{pmatrix}\right)$$

Soient $\lambda_1, \dots, \lambda_4 \in \mathbb{R}$

$$\lambda_1\begin{pmatrix} 1 \\ 0 \\ 0 \\ -1 \end{pmatrix} + \lambda_2\begin{pmatrix} 0 \\ 1 \\ 0 \\ -1 \end{pmatrix} + \lambda_3\begin{pmatrix} 0 \\ 0 \\ 1 \\ -1 \end{pmatrix} + \lambda_4\begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

$$\Rightarrow \begin{cases} \lambda_1 + \lambda_4 = 0 \\ \lambda_2 + \lambda_4 = 0 \\ \lambda_3 + \lambda_4 = 0 \\ -\lambda_1 - \lambda_2 - \lambda_3 + \lambda_4 = 0 \end{cases} \Rightarrow \begin{cases} \lambda_1 = -\lambda_4 \\ \lambda_2 = -\lambda_4 \\ \lambda_3 = -\lambda_4 \\ -\lambda_1 - \lambda_2 - \lambda_3 + \lambda_4 = 0 \end{cases} \Rightarrow \begin{cases} \lambda_1 = -\lambda_4 \\ \lambda_2 = -\lambda_4 \\ \lambda_3 = -\lambda_4 \\ \lambda_4 = 0 \end{cases}$$

F+H : ces vect. sont linéairemt indépendants, $\dim F+H = 4 \subseteq \mathbb{R}^4$. Soit $F+H = \mathbb{R}^4$.