

② a

$$\begin{array}{cccc|ccc} 1 & 1 & -1 & -5 & 0 & \boxed{2} & -3 \\ -2 & 1 & 0 & 1 & 0 & \leftarrow & \\ 3 & -3 & 1 & 3 & 0 & \leftarrow & \\ 1 & -2 & 1 & 4 & 0 & \leftarrow & \end{array} \quad -1$$

$$\begin{array}{cccc|ccc} 1 & 1 & -1 & -5 & 0 & & \\ 0 & 3 & -2 & -9 & 0 & \boxed{2} & 1 \\ 0 & -6 & 4 & 18 & 0 & \leftarrow & \\ 0 & -3 & 2 & 9 & 0 & \leftarrow & \end{array}$$

$$\begin{array}{cccc|ccc} 1 & 1 & -1 & -5 & 0 & & \\ 0 & 3 & -2 & -9 & 0 & & \\ 0 & 0 & 0 & 0 & 0 & & \\ 0 & 0 & 0 & 0 & 0 & & \end{array}$$

$(\Rightarrow \text{rg}(A) = 2)$

$$\begin{array}{c} \uparrow \quad \uparrow \\ \underline{x_3 = \lambda_1} \quad \underline{x_4 = \lambda_2} \end{array}$$

$$3x_2 = 2x_3 + 9x_4 \Rightarrow \underline{x_2 = \frac{2}{3}\lambda_1 + 3\lambda_2}$$

$$x_1 = -x_2 + x_3 + 5x_4 = -\frac{2}{3}\lambda_1 - 3\lambda_2 + \lambda_1 + 5\lambda_2$$

$$\Rightarrow \underline{x_1 = \frac{1}{3}\lambda_1 + 2\lambda_2}$$

$$\Rightarrow \vec{x} = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} = \lambda_1 \begin{pmatrix} +\frac{1}{3} \\ \frac{2}{3} \\ 1 \\ 0 \end{pmatrix} + \lambda_2 \begin{pmatrix} 2 \\ 3 \\ 0 \\ 1 \end{pmatrix}$$

$$\Rightarrow \ker A = \text{Vect} \left(\begin{pmatrix} 1 \\ 2 \\ 3 \\ 0 \end{pmatrix}, \begin{pmatrix} 2 \\ 3 \\ 0 \\ 1 \end{pmatrix} \right)$$

$$\dim(\ker A) = 2$$