Base $\left(\begin{pmatrix}1\\1\\1\end{pmatrix}, \begin{pmatrix}1\\0\\-1\end{pmatrix}, \begin{pmatrix}0\\1\\1\end{pmatrix}\right)$ de $\mathbb{R}^3$

$\quad e_1 \quad e_2 \quad e_3$

Base duale (e_1^*, e_2^*, e_3^*)

$e_1^*\left(\begin{pmatrix}x\\y\\z\end{pmatrix}\right) = \lambda x + \mu y + \nu z$ avec λ, μ, ν à déterminer

<u>Th</u> : $e_i^*(e_j) = \delta_{ij}$

On doit avoir

- $e_1^*(e_1) = \lambda + \mu + \nu = 1 \quad (L_1)$

$e_1^*(e_2) = \lambda - \nu = 0 \quad (L_2)$

$e_1^*(e_3) = \mu + \nu = 0 \quad (L_3)$

$(L_1) - (L_3) \quad \lambda = 1 \qquad (L_2) \quad \nu = 1 \qquad (L_3) \quad \mu = -1$

d'où $e_1^*(x, y, z) = x - y + z$

- e_2^* ?

$$\begin{cases} \lambda + \mu + \nu = 0 \\ \lambda - \nu = 1 \\ \mu + \nu = 0 \end{cases} \rightsquigarrow \begin{array}{l} \lambda = 0, \ \nu = -1 \\ \mu = 1 \end{array}$$

$e_2^*(x, y, z) = y - z$

- e_3^* ?

Exo : (f_1, f_2, f_3) base de $\mathbb{R}^{3*}$

$f_1(x,y,z) = x - y$ $\quad$ $f_2(x,y,z) = x + y - z$

$f_3(x,y,z) = x + z$

Base contraduale (e_1, e_2, e_3)

- $e_1 = \begin{pmatrix} \lambda \\ \mu \\ \nu \end{pmatrix} \in \mathbb{R}^3$

$$f_1(e_1) = \lambda - \mu = 1$$
$$f_2(e_1) = \lambda + \mu - \nu = 0 \quad \rightsquigarrow$$
$$f_3(e_1) = \lambda + \nu = 0$$