Décomposition LU de $\begin{array}{c} \begin{pmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{pmatrix} \end{array} \begin{array}{l} L_1 \\ L_2 \\ L_3 \end{array}$

$L_1 \leftarrow \frac{1}{2} L_1 \qquad U = \begin{pmatrix} 1 & -1/2 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{pmatrix} \qquad L = \begin{pmatrix} 2 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$

$L_2 \leftarrow L_2 + L_1 \qquad U = \begin{pmatrix} 1 & -1/2 & 0 \\ 0 & 3/2 & -1 \\ 0 & -1 & 2 \end{pmatrix} \qquad L = \begin{pmatrix} 2 & 0 & 0 \\ -1 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix}$

$L_2 \leftarrow \frac{2}{3} L_2 \qquad U = \begin{pmatrix} 1 & -1/2 & 0 \\ 0 & 1 & -2/3 \\ 0 & -1 & 2 \end{pmatrix} \qquad L = \begin{pmatrix} 2 & 0 & 0 \\ -1 & 3/2 & 0 \\ 0 & 0 & 0 \end{pmatrix}$

$L_3 \leftarrow L_3 + L_2 \qquad \ldots$

$$U = \begin{pmatrix} 1 & -1/2 & 0 \\ 0 & 1 & -2/3 \\ 0 & 0 & 1 \end{pmatrix}, \quad L = \begin{pmatrix} 2 & 0 & 0 \\ -1 & 3/2 & 0 \\ 0 & -1 & 4/3 \end{pmatrix}$$

Décomposition QU de $A = \begin{pmatrix} 12 & -51 & 4 \\ 6 & 167 & -68 \\ -4 & 24 & -41 \end{pmatrix}$

$a_1 = \begin{pmatrix} 12 \\ 6 \\ -4 \end{pmatrix} \quad a_2 = \begin{pmatrix} -51 \\ 167 \\ 24 \end{pmatrix} \quad a_3 = \begin{pmatrix} 4 \\ -68 \\ -41 \end{pmatrix}$

$\underline{GS}$:

$f_1 = a_1 \qquad \|f_1\|^2 = 196 = 14^2$

$$f_2 = a_2 - \frac{\langle f_1, a_2\rangle}{\|f_1\|^2} \cdot f_1 = \dots = \begin{pmatrix} -69 \\ 158 \\ 30 \end{pmatrix}$$

$\langle f_1, a_2\rangle = 294 \qquad \|f_2\|^2 = 30625 = 175^2$

$$f_3 = a_3 - \frac{\langle f_1, a_3\rangle}{\|f_1\|^2} \cdot f_1 - \frac{\langle f_2, a_3\rangle}{\|f_2\|^2} \cdot f_2 = \frac{1}{5}\begin{pmatrix} -58 \\ 6 \\ 165 \end{pmatrix}$$

$\langle f_1, a_3\rangle = -196 \qquad \langle f_2, a_3\rangle = -12250$

$\|f_3\|^2 = 1225 = 35^2$

$$Q = \frac{1}{175}\begin{pmatrix} 150 & -69 & -58 \\ 75 & 158 & 6 \\ -50 & 30 & -165 \end{pmatrix} \qquad R = \begin{array}{c} \\ e_1 \\ e_2 \\ e_3 \end{array}\begin{array}{c} \begin{array}{ccc} a_1 & a_2 & a_3 \end{array} \\ \begin{pmatrix} 14 & 21 & -14 \\ 0 & 175 & -70 \\ 0 & 0 & 35 \end{pmatrix} \end{array}$$