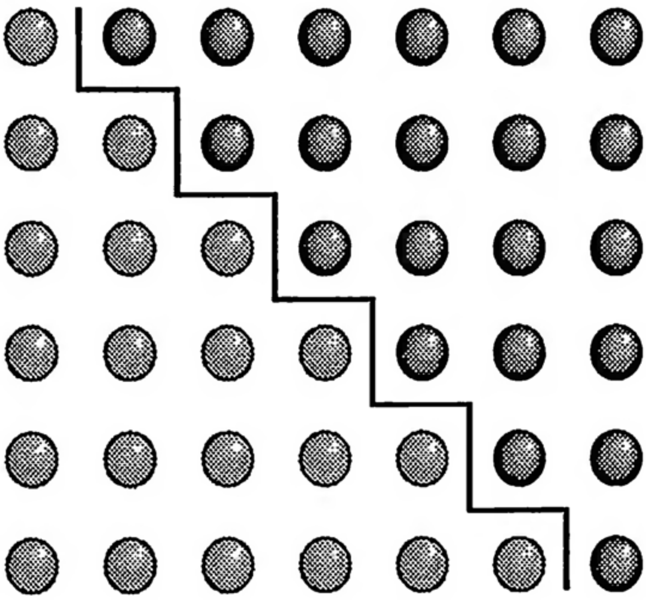
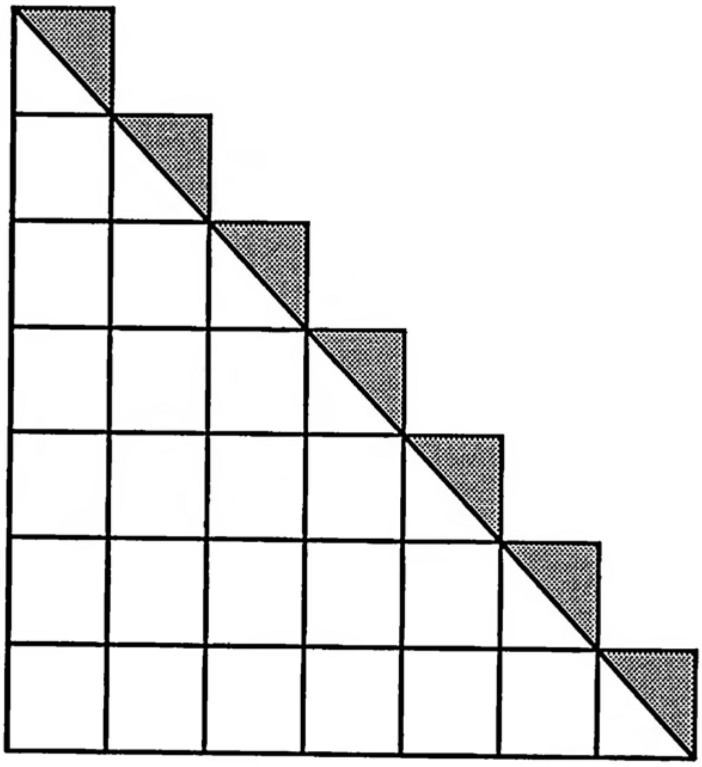
# Sums of Integers I



$$1 + 2 + \cdots + n = \frac{1}{2}n(n + 1)$$

—"The ancient Greeks"
(as cited by Martin Gardner)

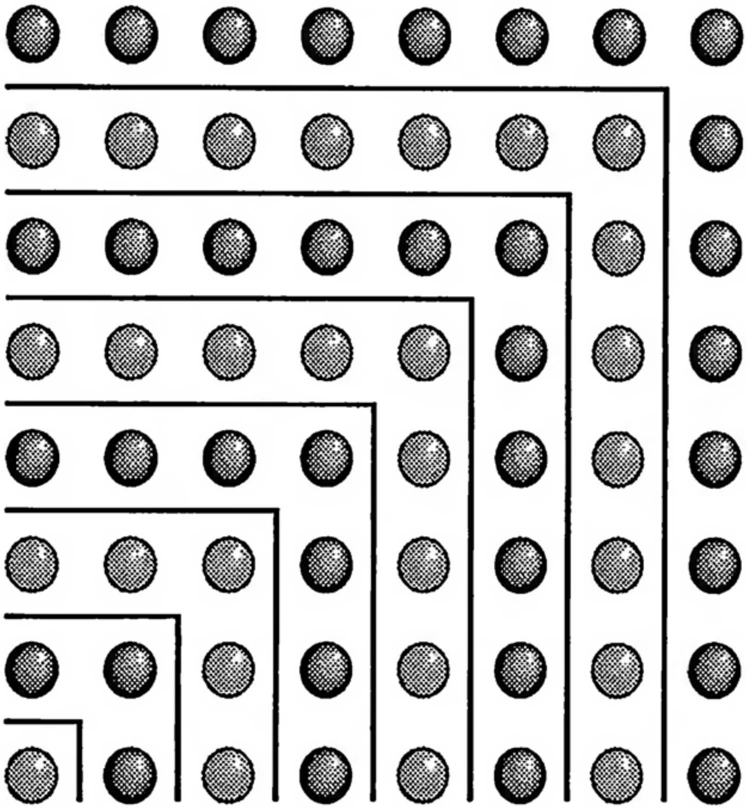
# Sums of Integers II



$$1 + 2 + \cdots + n = \frac{n^2}{2} + \frac{n}{2}$$

—Ian Richards

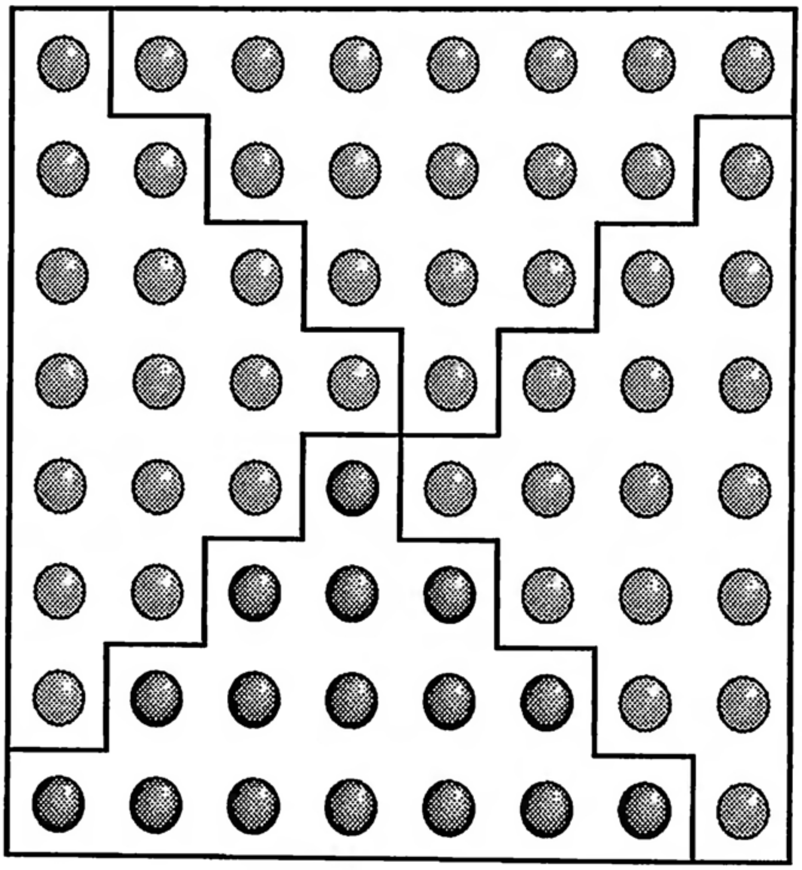
# Sums of Odd Integers I



$$1+3+5+\cdots+(2n-1) = n^2$$

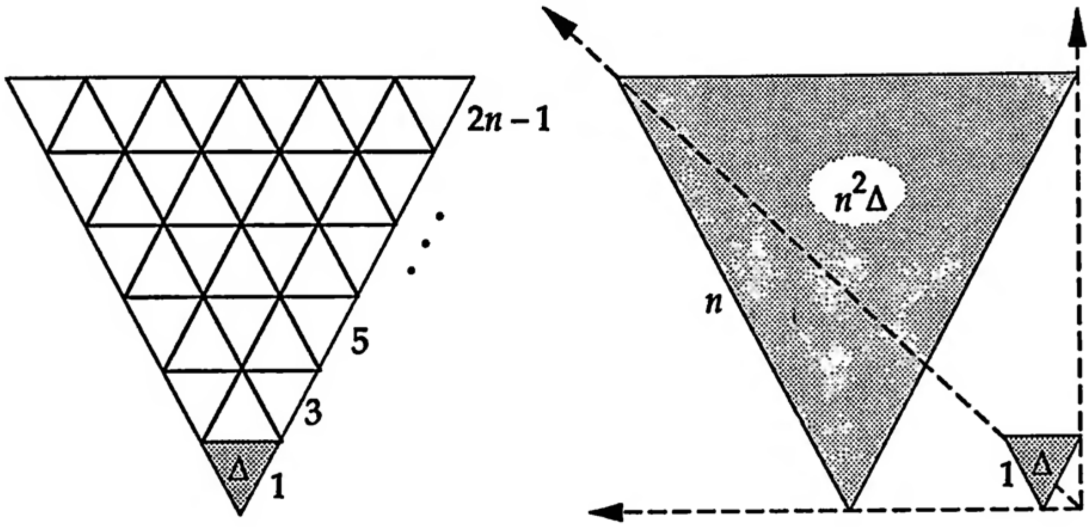
—Nicomachus of Gerasa (circa A. D. 100)

# Sums of Odd Integers II



$$1 + 3 + \cdots + (2n - 1) = \frac{1}{4}(2n)^2 = n^2$$

# Sums of Odd Integers III




$$\Delta + 3\cdot\Delta + \cdots + (2n-1)\cdot\Delta = A = n^2\cdot\Delta$$

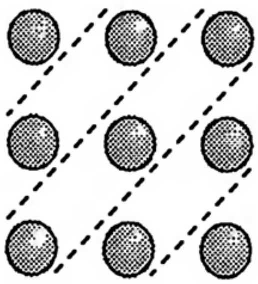
$$\sum_{i=1}^{n} (2i-1) = n^2$$

—Jenő Lehel

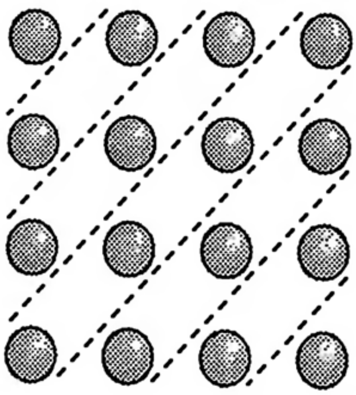
# Squares and Sums of Integers

I.

$$1+2+1=2^2$$



$$1+2+3+2+1 = 3^2$$



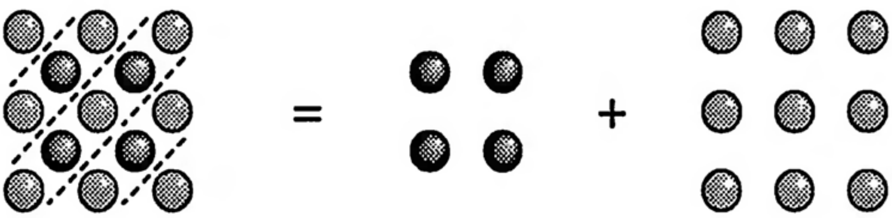
$$1+2+3+4+3+2+1 = 4^2$$

$$1+2+\cdots+(n-1)+n+(n-1)+\cdots+2+1 = n^2$$

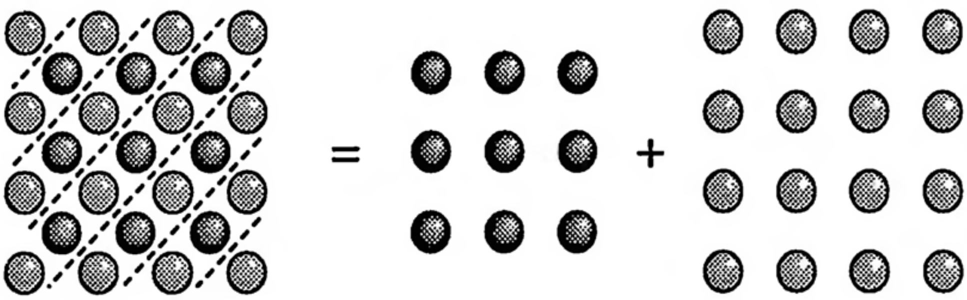
—"The ancient Greeks"
(as cited by Martin Gardner)

II.

$$1+3+1 = 1^2+2^2$$



$$1+3+5+3+1 = 2^2+3^2$$



$$1+3+5+7+5+3+1 = 3^2+4^2$$

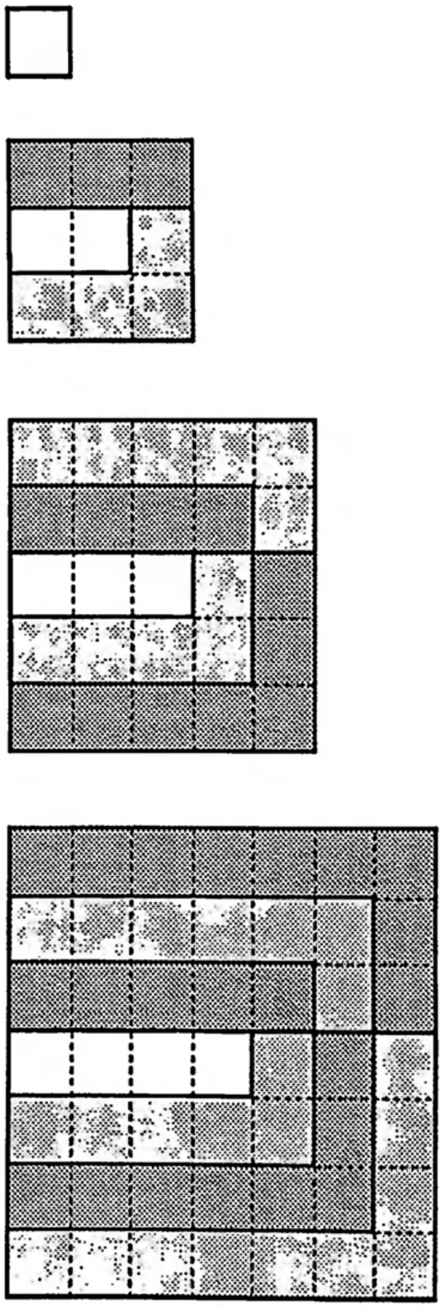
$$\vdots$$

$$1+3+\cdots+(2n-1)+(2n+1)+(2n-1)+\cdots+3+1 = n^2+(n+1)^2$$

—Hee Sik Kim

# Arithmetic Progressions with Sum Equal to the Square of the Number of Terms

$$\sum_{k=n}^{3n-2} k = (2n-1)^2; \; n = 1, 2, 3, \cdots .$$



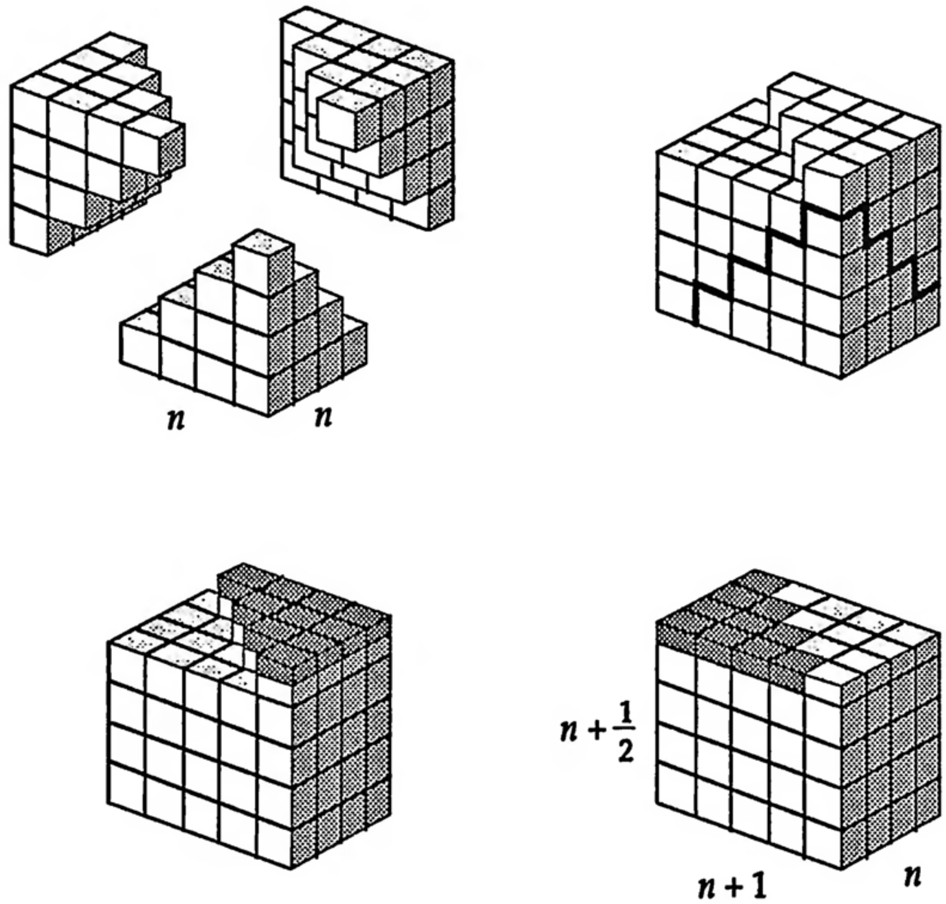
$n = 4$

$4 + 5 + 6 + 7 + 8 + 9 + 10 = 7^2$

—James O. Chilaka

# Sums of Squares I

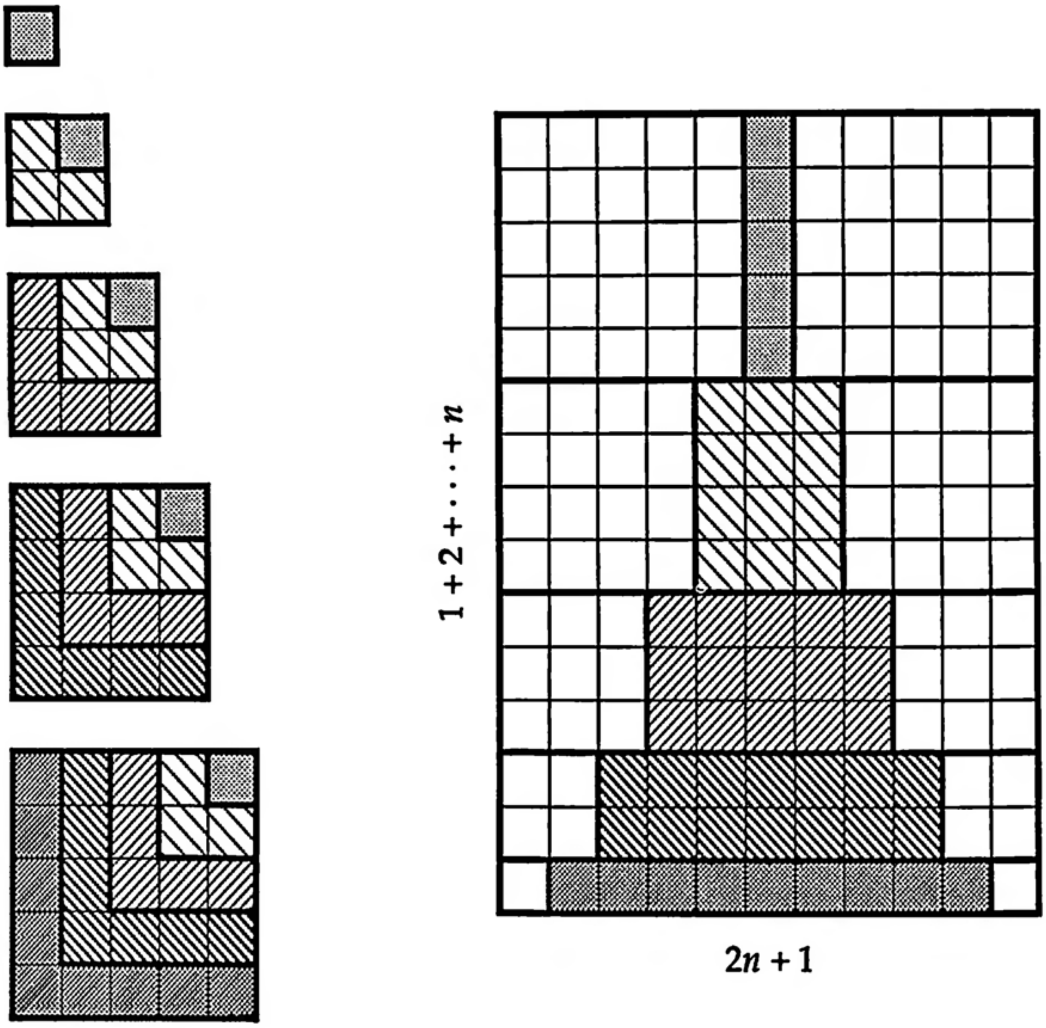
$$1^2 + 2^2 + \cdots + n^2 = \frac{1}{3}n(n+1)(n+\frac{1}{2})$$




—Man-Keung Siu

# Sums of Squares II

$$3(1^2+2^2+\cdots+n^2) = (2n+1)(1+2+\cdots+n)$$




—Martin Gardner and Dan Kalman
(independently)