Base duale de (e_1, e_2, e_3) de $\mathbb{R}^3$

avec $e_1 = \begin{pmatrix}1\\1\\1\end{pmatrix}$ $e_2 = \begin{pmatrix}1\\0\\-1\end{pmatrix}$ $e_3 = \begin{pmatrix}0\\1\\1\end{pmatrix}$

Disons (f_1, f_2, f_3)

Fait $f\begin{pmatrix}x\\y\\z\end{pmatrix} = \alpha_1 x + \alpha_2 y + \alpha_3 z$ où $\alpha_1, \alpha_2, \alpha_3 \in \mathbb{R}$

$f \in (\mathbb{R}^3)^*$ $= (\alpha_1, \alpha_2, \alpha_3)\begin{pmatrix}x\\y\\z\end{pmatrix}$

Pour calculer $f_1(x) = \alpha_1 x + \alpha_2 y + \alpha_3 z$, on utilise

par Th $\begin{cases} f_1(e_1) = 1 \\ f_1(e_2) = 0 \\ f_1(e_3) = 0 \end{cases}$ d'où $\begin{cases} \alpha_1 + \alpha_2 + \alpha_3 = 1 \\ \alpha_1 - \alpha_3 = 0 \\ \alpha_2 + \alpha_3 = 0 \end{cases}$

$\iff \begin{cases} \alpha_3 = \alpha_1 \\ \alpha_3 = -\alpha_2 \\ \alpha_3 - \alpha_3 + \alpha_3 = 1 \end{cases} \iff \begin{cases} \alpha_1 = 1 \\ \alpha_2 = -1 \\ \alpha_3 = 1 \end{cases}$

d'où $f_1(x, y, z) = x - y + z$

Calcul f_2 $\qquad f_2(x,y,z) = d_1 x + d_2 y + d_3 z$

$$\begin{cases} f_2(e_1) = 0 \\ f_2(e_2) = 1 \\ f_2(e_3) = 0 \end{cases} \iff \begin{cases} d_1 + d_2 + d_3 = 0 \\ d_1 - d_3 = 1 \\ d_2 + d_3 = 0 \end{cases}$$

...