

Gram-Schmidt $(a_1, \dots, a_n) \rightarrow (f_1, \dots, f_n)$ ortho.

$$\text{Vect}(a_1, \dots, a_k) = \text{Vect}(f_1, \dots, f_k)$$

$$f_k = a_k + \text{comb. } a_1, \dots, a_{k-1} \\ f_1, \dots, f_{k-1}$$

$$\bullet f_1 = a_1$$

$$\bullet f_2 = a_2 + d_1 f_1$$

$$\langle f_1, f_2 \rangle = 0 \Leftrightarrow \langle f_1, a_2 + d_1 f_1 \rangle = 0$$

$$\Leftrightarrow \langle f_1, a_2 \rangle + \overline{d_1} \|f_1\|^2 = 0$$

$$\Leftrightarrow \overline{d_1} = - \frac{\langle f_1, a_2 \rangle}{\|f_1\|^2}$$

$$f_2 = a_2 - \frac{\langle f_1, a_2 \rangle}{\|f_1\|^2} \cdot f_1$$

$$\bullet f_3 = a_3 + d_1 f_1 + d_2 f_2$$

$$\langle f_1, f_3 \rangle = 0 \Leftrightarrow \langle f_1, a_3 \rangle + \overline{d_1} \|f_1\|^2 + d_2 \langle f_1, f_2 \rangle$$

$$\Leftrightarrow \overline{d_1} = - \frac{\langle f_1, a_3 \rangle}{\|f_1\|^2}$$

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