$$q(x) = \boxed{x_1^2 - 2x_1x_2 + 4x_1x_3 - 2x_1x_4} + 5x_2^2 - 8x_2x_3 + 14x_2x_4 + 5x_3^2 - 8x_3x_4 + 10x_4^2$$

$$x_1^2 + 2x_1\underbrace{(-x_2 + 2x_3 - x_4)}$$

$$= (x_1 - x_2 + 2x_3 - x_4)^2 - \left[x_2^2 + 4x_3^2 + x_4^2 - 4x_2x_3 + 2x_2x_4 - 4x_3x_4\right]$$

$$\boxed{\begin{aligned} &x^2 + 2xa \\ &= (x+a)^2 - a^2 \end{aligned}}$$

$$\rightsquigarrow q(x) = (x_1 - x_2 + 2x_3 - x_4)^2 + \boxed{4x_2^2 - 4x_2x_3 + 12x_2x_4} + \underbrace{x_3^2 - 4x_3x_4 + 9x_4^2}$$

$$4x_2^2 - 4x_2x_3 + 12x_2x_4 = 4\left[x_2^2 + 2x_2\left(-\frac{1}{2}x_3 + \frac{3}{2}x_4\right)\right]$$

$$= 4\left[\left(x_2 - \frac{1}{2}x_3 + \frac{3}{2}x_4\right)^2 - \underbrace{\left(\frac{1}{4}x_3^2 + \frac{9}{4}x_4^2 - \frac{3}{2}x_3x_4\right)}\right]$$

$$\rightsquigarrow q(x) = (x_1 - x_2 + 2x_3 - x_4)^2 + 4\underbrace{\left(x_2 - \frac{1}{2}x_3 + \frac{3}{2}x_4\right)}^2 + 2x_3x_4$$

$$uv = \frac{1}{4}\left[(u+v)^2 - (u-v)^2\right]$$

$$q(x) = \overbrace{(x_1 - x_2 + 2x_3 - x_4)}^{f_1}{}^2 + 4\overbrace{\left(x_2 - \tfrac{1}{2}x_3 + \tfrac{3}{2}x_4\right)}^{f_2}{}^2 + \frac{1}{2}\overbrace{(x_3 + x_4)}^{f_3}{}^2 - \frac{1}{2}\overbrace{(x_3 - x_4)}^{f_4}{}^2$$

Dans la base orthogonale de (f_1, f_2, f_3, f_4), la matrice de q est

$$\begin{pmatrix} 1 & & & 0 \\ & 4 & & \\ & & 1/2 & \\ 0 & & & -1/2 \end{pmatrix}$$

signature $q = (3, 1)$

nbre de coeff > 0

nbre de coeff. < 0

$$q(x) = \boxed{x_1x_2 + 2x_1x_3 - x_1x_4 + x_2x_3 + 3x_2x_4} + 9x_3x_4$$

$$x_1x_2 + x_1\overbrace{(2x_3 - 4x_4)}^{a} + x_2\underbrace{(x_3 + 3x_4)}_{b}$$

$$uv + ua + vb = (u+b)(v+a) - ab$$

$$(x_1 + x_3 + 3x_4)(x_2 + 2x_3 - x_4) - 2x_3^2 + 6x_3x_4 - 4x_3x_4 - 12x_4^2$$

$\rightsquigarrow \quad q(x) = \underbrace{\frac{1}{4}}(x_1 + x_2 + 3x_3 + 2x_4)^2$

$\quad - \underbrace{\frac{1}{4}}(x_1 - x_2 - x_3 + 4x_4)^2 \quad \underbrace{-2x_3^2 + 4x_3x_4 + 3x_4^2}$

$$\underbrace{-2}(x_3 - x_4)^2 + \underbrace{5}x_4^2$$

$$\text{signature}(q) = (2, 2)$$

---

dim 3

$$q(x) = (x_1 + x_2)^2 + x_2^2 \underbrace{+ 0 \times x_3^2}$$

ajouté pour avoir une base