$\phi(x,y) = \dots$

1. $\phi(z,w) = 2\times 5 + (-1)\times 15 + \dots$

2.

$$\phi(x,y) = x_1(y_1 + 6y_3) + x_2(y_2 + 2y_1 - 3y_3) + x_3(3y_3 + 3y_1 + y_2)$$

$$= (x_1, x_2, x_3)\begin{pmatrix} y_1 + 6y_3 \\ y_2 + 2y_1 - 3y_3 \\ 3y_3 + 3y_1 + y_2 \end{pmatrix}$$

$$= \underset{\underset{x^t}{\shortparallel}}{(x_1, x_2, x_3)} \begin{array}{c|ccc} & y_1 & y_2 & y_3 \\ \hline x_1 & 1 & 0 & 6 \\ x_2 & 2 & 1 & -3 \\ x_3 & 3 & 1 & 3 \end{array} \begin{pmatrix} y_1 \\ y_2 \\ y_3 \end{pmatrix} \doteq y$$

coeff de $x_1 y_3$

coeff. de $x_3 y_2$

$$= x^t A y$$

Noyau (à gauche) de ϕ

$x \in \mathbb{R}^3$ tq $\forall y \in \mathbb{R}^3$ $\phi(x,y) = 0$

$$y_1(x_1 + 2x_2 + 3x_3) + y_2(x_2 + x_1) + y_3(3x_3 + 6x_1 - 3x_2) = 0$$

$\forall y_1, y_2, y_3 \in \mathbb{R}$

$\Updownarrow$

$$y_1=1,\ y_2=y_3=0 \Rightarrow \begin{cases} x_1+2x_2+3x_3=0 & L_1 \\ x_2+x_3=0 & L_2 \\ 3x_3+6x_1-3x_2=0 \leftarrow L_3-6L_1 & L_3 \end{cases}$$

$y_2=1,\ y_1=y_3=0 \Rightarrow$

$y_3=1,\ y_1=y_2=0 \Rightarrow$

$$\begin{cases} x_1+2x_2+3x_3=0 \\ x_2+x_3=0 \\ -\cancel{15x_2-15x_3=0} \end{cases} \times -15$$

$$\begin{cases} x_1+2x_2+3x_3=0 \\ x_2+x_3=0 \end{cases}$$

Th : solutions sont un sev de dim. $3-2=1$

$$\operatorname{Ker}\phi = \operatorname{Vect}\left(\begin{pmatrix}-1\\-1\\1\end{pmatrix}\right)$$

$x_1 = -2x_2-3x_3$

$= \cancel{2-3=1}$

$$\operatorname{rang}\phi = 3-\dim\operatorname{Ker}\phi$$
$$=2$$

3.

$\phi(v_i, v_j)$ $\forall i,j$

4. Orthogonal v_1 : $y\in\mathbb{R}^3$ tq $v_1 \perp y$

$$\Leftrightarrow \phi(v_1,y)=0 \Leftrightarrow y_1+y_2+3y_3+6y_3+2y_1-3y_3+3y_1+y_2=0 \ldots$$

$F = \mathrm{Vect}(v_1, v_2)$ Calculer $F^{\perp}$

$$\phi(v_2, y) = 0 \iff \boxed{a_1 y_1 + a_2 y_2 + a_3 y_3 = 0}$$

$$\begin{cases} \phi(v_1, y) = 0 \\ \phi(v_2, y) = 0 \end{cases} \leftarrow \text{solutions} = F^{\perp}$$