

$$\phi(x, y) = \dots$$

$$1. \quad \phi(z, w) = 2 \times 5 + (-2) \times 15 + \dots$$

2.

$$\phi(x, y) = x_1(y_1 + 6y_3) + x_2(y_2 + 2y_1 - 3y_3)$$

$$+ x_3(3y_3 + 3y_1 + y_2)$$

$$= (x_1, x_2, x_3) \begin{pmatrix} y_1 + 6y_3 \\ y_2 + 2y_1 - 3y_3 \\ 3y_3 + 3y_1 + y_2 \end{pmatrix}$$

$$= \underset{x^t}{(x_1, x_2, x_3)} \begin{pmatrix} \overset{y_1}{1} & \overset{y_2}{0} & \overset{y_3}{6} \\ \overset{y_1}{2} & \overset{y_2}{1} & \overset{y_3}{-3} \\ \overset{y_1}{3} & \overset{y_2}{1} & \overset{y_3}{3} \end{pmatrix} \begin{pmatrix} y_1 \\ y_2 \\ y_3 \end{pmatrix} = y$$

→ coeff de $x_1 y_3$

← coeff. de $x_3 y_2$

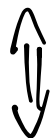
$$= x^t A y$$

Noyaux (à gauche) de ϕ

$$x \in \mathbb{R}^3 \text{ et } \forall y \in \mathbb{R}^3 \quad \phi(x, y) = 0$$

$$y_1(x_1 + 2x_2 + 3x_3) + y_2(x_2 + x_1) + y_3(3x_3 + 6x_1 - 3x_2) = 0$$

$$\forall y_1, y_2, y_3 \in \mathbb{R}$$



$$\begin{array}{l}
 y_1 = 1, y_2 = y_3 = 0 \Rightarrow \\
 y_2 = 1, y_1 = y_3 = 0 \Rightarrow \\
 y_3 = 1, y_1 = y_2 = 0 \Rightarrow
 \end{array}
 \Rightarrow \left\{ \begin{array}{l}
 x_1 + 2x_2 + 3x_3 = 0 \quad L_1 \\
 x_2 + x_3 = 0 \quad L_2 \\
 3x_3 + 6x_1 - 3x_2 = 0 \quad L_3
 \end{array} \right. \leftarrow L_3 - 6L_1$$

$$\left\{ \begin{array}{l}
 x_1 + 2x_2 + 3x_3 = 0 \\
 x_2 + x_3 = 0 \\
 -15x_2 - 15x_3 = 0
 \end{array} \right. \xrightarrow{x-15}$$

$$\left\{ \begin{array}{l}
 x_1 + 2x_2 + 3x_3 = 0 \\
 x_2 + x_3 = 0
 \end{array} \right.$$

\mathcal{H} = solutions sont un
 cas de dim. $3 - 2 = 1$

$$\text{Ker } \phi = \text{Vect} \left(\begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix} \right)$$

$$\begin{aligned}
 x_1 &= -2x_2 - 3x_3 \\
 &= \cancel{2-3} \dots
 \end{aligned}$$

$$\begin{aligned}
 \text{rang } \phi &= 3 - \dim \text{Ker } \phi \\
 &= 2
 \end{aligned}$$

3. $\phi(v_i, v_j) \neq i, j$

4. Orthogonal v_i : $y \in \mathbb{R}^3$ tq $v_i \perp y$

$$\Leftrightarrow \phi(v_1, y) = 0 \Leftrightarrow \left(y_1 + y_2 + 3y_3 + 6y_3 + 2y_1 - 3y_3 + 3y_1 + y_2 = 0 \dots \right)$$

$F = \text{Vect}(v_1, v_2)$ Calculer F^\perp

$$\phi(v_2, y) = 0 \Leftrightarrow a_1 y_1 + a_2 y_2 + a_3 y_3 = 0$$

$$\left\{ \begin{array}{l} \phi(v_1, y) = 0 \\ \phi(v_2, y) = 0 \end{array} \right.$$

← solutions = F^\perp