

Decomposition LU

$$A = \begin{pmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{pmatrix}$$

$$L_1 \leftarrow \frac{1}{2} L_1 \quad U = \begin{pmatrix} 1 & -\frac{1}{2} & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{pmatrix} \quad L = \begin{pmatrix} 2 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$L_2 \leftarrow L_2 + L_1 \quad U = \begin{pmatrix} 1 & -\frac{1}{2} & 0 \\ 0 & \frac{3}{2} & -1 \\ 0 & -1 & 2 \end{pmatrix} \quad L = \begin{pmatrix} 2 & 0 & 0 \\ -1 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$L_2 \leftarrow \frac{2}{3} L_2 \quad U = \begin{pmatrix} 1 & -\frac{1}{2} & 0 \\ 0 & 1 & -\frac{2}{3} \\ 0 & -1 & 2 \end{pmatrix} \quad L = \begin{pmatrix} 2 & 0 & 0 \\ -1 & \frac{3}{2} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$L_3 \leftarrow L_3 + L_2 \quad U = \begin{pmatrix} 1 & -\frac{1}{2} & 0 \\ 0 & 1 & -\frac{2}{3} \\ 0 & 0 & \frac{4}{3} \end{pmatrix} \quad L = \begin{pmatrix} 2 & 0 & 0 \\ -1 & \frac{3}{2} & 0 \\ 0 & -1 & 1 \end{pmatrix}$$

$$L_3 \leftarrow \frac{3}{4} L_3$$

$$U = \begin{pmatrix} 1 & -\frac{1}{2} & 0 \\ 0 & 1 & -\frac{2}{3} \\ 0 & 0 & 1 \end{pmatrix}$$

$$L = \begin{pmatrix} 2 & 0 & 0 \\ -1 & \frac{3}{2} & 0 \\ 0 & -1 & \frac{4}{3} \end{pmatrix}$$

