

Fonction φ d'Euler

$$\varphi(n) = |(\mathbb{Z}/n\mathbb{Z})^*|$$

$$\text{Si } \text{PGCD}(n, m) = 1, \quad (\mathbb{Z}/nm\mathbb{Z})^* \simeq (\mathbb{Z}/n\mathbb{Z})^* \times (\mathbb{Z}/m\mathbb{Z})^*$$

$$\text{Si } (n, m) = 1, \quad \varphi(nm) = \varphi(n)\varphi(m)$$

$$\text{Si } p \text{ premier } \varphi(p) = p-1 = \text{card} \left\{ 0 \leq a \leq p-1 \mid \begin{array}{l} a \neq 0 \\ (a, p) = 1 \end{array} \right\}$$

$$\begin{aligned} \text{plus g\u00e9n\u00e9ralis\u00e9} \\ e \geq 1 \quad \varphi(p^e) &= p^{e-1}(p-1) \\ &= \text{card} \left\{ 0 \leq a \leq p^e - 1 \mid \right. \\ &\quad \left. (a, p^e) = 1 \right\} \end{aligned}$$

$p \nmid a$

$$= p^e - \underbrace{\text{card } \{0 \leq a \leq p^e - 1\}}_{p/a} \frac{1}{p}$$

...

Calculus

$$\varphi(n) = n \prod_{p|n} \left(1 - \frac{1}{p}\right)$$