Corrigé CC9Nov2011

Ex1.

$$\left(\begin{array}{cccc|c} \boxed{3} & 2 & 2 & 2 & 2 \\ 2 & 3 & 2 & 5 & 3 \\ 9 & 1 & 4 & -5 & 1 \\ 2 & 2 & 3 & 4 & 5 \\ 7 & 1 & 6 & -1 & 7 \end{array}\right) \begin{array}{l} \cdot(-2),\ \cdot(-3),\ \cdot(-2),\ \cdot(-7) \\ /\cdot 3 \ + \\ + \\ /\cdot 3 \ + \\ /\cdot 3 \ + \end{array}$$

$$\left(\begin{array}{cccc|c} 3 & 2 & 2 & 2 & 2 \\ 0 & \boxed{5} & 2 & 11 & 5 \\ 0 & -5 & -2 & -11 & -5 \\ 0 & 2 & 5 & 8 & 11 \\ 0 & -11 & 4 & -17 & 7 \end{array}\right) \begin{array}{l} \\ \cdot(-2),\ \cdot 11 \\ + \\ /\cdot 5 \ + \\ /\cdot 5 \ + \end{array}$$

$$\left(\begin{array}{cccc|c} 3 & 2 & 2 & 2 & 2 \\ 0 & 5 & 2 & 11 & 5 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 21 & 18 & 45 \\ 0 & 0 & 42 & 36 & 90 \end{array}\right) \begin{array}{l} \\ \\ \\ \cdot(-2) \\ + \end{array}$$

$$\left(\begin{array}{cccc|c} 3 & 2 & 2 & 2 & 2 \\ 0 & 5 & 2 & 11 & 5 \\ 0 & 0 & 21 & 18 & 45 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right) \Rightarrow \boxed{\text{rang} = 3}$$

$\boxed{x_4 = \lambda}$, $\quad 21 \cdot x_3 + 18 \cdot \lambda = 45 \Rightarrow \boxed{x_3 = \frac{15 - 6\lambda}{7}}$

$$5x_2 + 2x_3 + 11\lambda = 5x_2 + 2 \cdot \frac{15-6\lambda}{7} + 11\lambda = 5 \Rightarrow$$

$$\Rightarrow 5x_2 + \frac{30}{7} + \frac{65}{7}\lambda = 5 \Rightarrow \boxed{x_2 = \frac{1}{7} - \frac{13}{7}\lambda} \qquad \boxed{x_1 = -\frac{6}{7} + \frac{8}{7}\lambda}$$

$$3x_1 + 2 \cdot \left(\frac{1}{7} - \frac{13}{7}\lambda\right) + 2 \cdot \left(\frac{15-6\lambda}{7}\right) + 2\lambda = 2 \Rightarrow 3x_1 + \frac{18}{7} - \frac{24}{7}\lambda = 0$$

Alors les solutions ont la forme

$$\vec{x} \equiv \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} = \frac{1}{7} \cdot \begin{pmatrix} -6 \\ 1 \\ 15 \\ 0 \end{pmatrix} + \underbrace{\lambda \cdot \frac{1}{7}}_{\tilde{\lambda}} \cdot \begin{pmatrix} 8 \\ -13 \\ -6 \\ 7 \end{pmatrix}, \quad \lambda \in \mathbb{R}$$

ou également

$$\boxed{\vec{x} = \frac{1}{7} \cdot \begin{pmatrix} -6 \\ 1 \\ 15 \\ 0 \end{pmatrix} + \tilde{\lambda} \cdot \begin{pmatrix} 8 \\ -13 \\ -6 \\ 7 \end{pmatrix}, \quad \tilde{\lambda} \in \mathbb{R}}$$

<u>Remarques:</u> On a resolu $A \cdot \vec{x} = \vec{b}$ avec

$$A = \begin{pmatrix} 3 & 2 & 2 & 2 \\ 2 & 3 & 2 & 5 \\ 9 & 1 & 4 & -5 \\ 2 & 2 & 3 & 4 \\ 7 & 1 & 6 & -1 \end{pmatrix} \in \mathcal{M}_{5,4}(\mathbb{R}) \quad \text{et} \quad \vec{b} = \begin{pmatrix} 2 \\ 3 \\ 1 \\ 5 \\ 7 \end{pmatrix} \in \mathbb{R}^5$$

Soit $f: \mathbb{R}^4 \to \mathbb{R}^5, \vec{x} \mapsto A \cdot \vec{x}$. On a trouvé que $\mathrm{im}(f) \equiv \mathrm{im}(A)$ est un sous-espace vectoriel de dimension $\underline{3}$ dans $\mathbb{R}^5$, $\vec{b} \in \mathrm{im}(f)$, et que

$$\ker(f) \equiv \ker A = \mathrm{Vect}\left(\begin{pmatrix} 8 \\ -13 \\ -6 \\ 7 \end{pmatrix}\right) \equiv \left\{\mu \cdot \begin{pmatrix} 8 \\ -13 \\ -6 \\ 7 \end{pmatrix}, \mu \in \mathbb{R}\right\} \subset \mathbb{R}^4.$$

(Cf. aussi thm du rang : $\underbrace{\mathrm{rang}\, A}_{3} = 4 - \underbrace{\dim(\ker A)}_{1}$ ✓)