(b) $\left.\begin{array}{l}\dim(\operatorname{im} A) = \operatorname{rg} A = 2 \quad (\text{cf. ci-dessus}) \\ \underbrace{\begin{pmatrix}1\\-2\\3\\1\end{pmatrix}, \begin{pmatrix}-1\\0\\1\\1\end{pmatrix}}_{\mathcal{F} \text{ libre}} \in \operatorname{im} A\end{array}\right\} \Rightarrow \mathcal{F} \text{ une base de } \operatorname{im} A$

(c) $\ker f + \operatorname{im} f \equiv \ker A + \operatorname{im} A = \mathbb{R}^4$

$$\Updownarrow$$

$$\Big(\underbrace{\begin{pmatrix}1\\2\\3\\0\end{pmatrix}, \begin{pmatrix}2\\3\\0\\1\end{pmatrix}}_{\text{base de } \ker A}, \underbrace{\begin{pmatrix}1\\-2\\3\\1\end{pmatrix}, \begin{pmatrix}-1\\0\\1\\1\end{pmatrix}}_{\text{base de } \operatorname{im} A}\Big) \text{ une base de } \mathbb{R}^4$$

$$\Updownarrow$$

$$\begin{vmatrix} 1 & 2 & 1 & -1 \\ 2 & 3 & -2 & 0 \\ 3 & 0 & 3 & 1 \\ 0 & 1 & 1 & 1 \end{vmatrix} \neq 0$$

reste a demontrer

Alternativement:

CM: $f \in \operatorname{End}(E)$

$\ker f + \operatorname{im} f = E \Leftrightarrow \ker f \cap \operatorname{im} f = \{0_E\}$

ici $E = \mathbb{R}^4$

Alors reste a demontrer que $\ker A \cap \operatorname{im} A = \left\{\begin{pmatrix}0\\0\\0\\0\end{pmatrix}\right\}$