

Décomposition QR  $A = \begin{pmatrix} 12 & -51 & 4 \\ 6 & 167 & -68 \\ -4 & 24 & -41 \end{pmatrix}$

$$a_1 = \begin{pmatrix} 12 \\ 6 \\ -4 \end{pmatrix} \quad a_2 = \begin{pmatrix} -51 \\ 167 \\ 24 \end{pmatrix} \quad a_3 = \begin{pmatrix} 4 \\ -68 \\ -41 \end{pmatrix}$$

Gramm - Schmidt

$$f_1 = a_1 \quad \langle f_1, f_1 \rangle = \|f_1\|^2 = {}^t a_1 \cdot a_1 = 196 = (14)^2$$

$$f_2 = a_2 - \frac{\langle f_1, a_2 \rangle}{\|f_1\|^2} \times f_1 = \begin{pmatrix} -69 \\ 158 \\ 30 \end{pmatrix} \quad \langle f_1, a_2 \rangle = 294$$

$$\|f_2\|^2 = 30625 = 175^2$$

$$f_3 = a_3 - \frac{\langle f_1, a_3 \rangle}{\|f_1\|^2} \times f_1 - \frac{\langle f_2, a_3 \rangle}{\|f_2\|^2} \times f_2 = \frac{1}{5} \begin{pmatrix} -58 \\ 6 \\ 165 \end{pmatrix}$$

$$\langle f_1, a_3 \rangle = -196 \quad \langle f_2, a_3 \rangle = -12250$$

$$\|f_3\|^2 = 1225 = 35^2$$

$$e_1 = \frac{1}{\|f_1\|} f_1 = \frac{1}{7} \begin{pmatrix} 6 \\ 3 \\ -2 \end{pmatrix}$$

$$e_2 = \frac{1}{\|f_2\|} f_2 = \frac{1}{175} \begin{pmatrix} -69 \\ 158 \\ 30 \end{pmatrix}$$

$$e_3 = \frac{1}{\|f_3\|} f_3 = \frac{1}{175} \begin{pmatrix} -58 \\ 6 \\ -165 \end{pmatrix} \quad \text{BON}$$

donc

$$Q = \frac{1}{175} \begin{pmatrix} e_1 & e_2 & e_3 \\ 150 & -69 & -58 \\ 75 & 158 & 6 \\ -50 & 30 & -165 \end{pmatrix}$$

$$\langle e_i, a_j \rangle$$

$$i \leq j$$

$$R = \begin{pmatrix} 14 & 21 & -14 \\ 0 & 175 & -70 \\ 0 & 0 & 35 \end{pmatrix}$$

$$\langle e_1, a_2 \rangle = \frac{1}{\|f_1\|} \langle f_1, a_2 \rangle = \frac{1}{14} \times 294 \dots$$